In Table 1 we give the values of the successive approximations for the stress components in the form

$$\sigma_{rn} = \frac{1}{p}(\sigma_r^\circ + \sigma_r^1 + \ldots + \sigma_r^n), \qquad \sigma_{\varphi n} = \frac{1}{p}(\sigma_\varphi^\circ + \sigma_\varphi^1 + \ldots + \sigma_\varphi^n)$$

for the values $\varphi = 0$, $q = -1.6$, $\nu = 0.3$, which correspond to the values of the parameters in [1]. This allows us to compare the results.

Thus, the presence of nonhomogeneity implies the formation of shear stresses, although insignificant in magnitude. In addition a quantitative variation of the maximal values of σ_r by 9% and of σ_φ by 30%, is observed.

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ON THE MAGNETOELASTICITY OF THIN SHELLS AND PLATES

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We consider some problems of magnetoelastic oscillations of thin electrically conducting plates and shells situated in a stationary magnetic field. On this basis of the solutions, obtained by the method of asymptotic integration of the three-dimensional equations of magnetoelasticity, we formulate a hypothesis relative to the character of the variation of the electromagnetic field and of the elastic displacements along the thickness of the shell. This allows us to reduce the three-dimensional equations of magnetoelasticity to two-dimensional ones, which facilitates in an essential way the study of the magnetoelastic problems of thin bodies.

The problem of the investigation of magnetoelastic oscillations of electrically conducting shells in a magnetic field reduces to the simultaneous solution of the equations of magnetoelasticity in the domain occupied by the shell and the equations of the electrodynamics in the exterior of the shell. The equations of the magnetoelasticity consist of the equations of the motion of the elastic medium taking into account forces of electromagnetic origin (the Lorentz forces), and the equations of the electrodynamics of a moving, electrically-conducting medium [1 – 3].

1. We assume that an isotropic thin shell of constant thickness $2h$, made of a material with finite electrical conductivity, is situated in a stationary magnetic field with a given vector of the magnetic induction. We assume that the magnetic and dielectric permeabilities of the medium surrounding the shell are equal to one, i. e. we assume that the shell is situated in a vacuum.

The elastic and the electromagnetic properties of the material are characterized by the modulus of elasticity E, Poisson's ratio ν, the density ρ, electric conductivity σ, magnetic permeability μ, dielectric permeability ε. We assume that none of these quantities depend on the coordinates, time and the electromagnetic field.

We assume that the shell is given in a Cartesian system of coordinates α, β, γ (α and β coincide with the lines of curvature of the median surface). The geometry of the median surface of the shell is such that the coefficients $A(\alpha, \beta)$, $B(\alpha, \beta)$ of the first quadratic form and the principal curvatures $k_1(\alpha, \beta)$, $k_2(\alpha, \beta)$ are either constants or at differentiation they behave as constants with the required degree of accuracy [4, 6]. We also assume that the elastic displacements of the shell and the electromagnetic perturbations are so small that for the investigation of the problem under consideration we may use the linear equations.

At the same time we assume that the problem of the magnetostatics for the unexcited state is solved. The vectors $\mathbf{B}_0^{(e)}(B_{0\alpha}^{(e)}, B_{0\beta}^{(e)}, B_{0\gamma}^{(e)})$ and $\mathbf{B}_0^{(i)}(B_{0\alpha}^{(i)}, B_{0\beta}^{(i)}, B_{0\gamma}^{(i)})$ of the magnetic induction for the exterior and the interior domains, respectively, are known, i. e. we assume that $\mathbf{B}_0^{(e)}$ and $\mathbf{B}_0^{(i)}$ satisfy the known equations and the conditions at the separation surface of the two domains [1, 2]

$$\operatorname{rot} \mathbf{H}_0 = 0, \qquad \operatorname{div} \mathbf{B}_0 = 0 \tag{1.1}$$

$$[\mathbf{B}_0^{(e)} - \mathbf{B}_0^{(i)}] \cdot \mathbf{n} = 0, \qquad [\mathbf{H}_0^{(e)} - \mathbf{H}_0^{(i)}] \cdot \mathbf{n} = 0 \tag{1.2}$$

Here $\mathbf{H}_0^{(e)} = \mathbf{B}_0^{(e)}$, $\mathbf{H}_0^{(i)} = \mu^{-1} \mathbf{B}_0^{(i)}$ are the intensity vectors of the magnetic field corresponding to the exterior and the interior domains, $\mathbf{n}$ is the vector normal to the surface of the shell (interface).

The equations of the electrodynamics for the moving medium takes the form [1, 2]:

in the interior domain (inside the body of the shell)

$$\operatorname{rot} \mathbf{H}^{(i)} = \frac{4\pi\sigma}{c}\left[\mathbf{E}^{(i)} + \frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} \times \mathbf{B}^{(i)}\right] + \frac{1}{c}\frac{\partial \mathbf{D}^{(i)}}{\partial t}$$

$$\operatorname{rot} \mathbf{E}^{(i)} = -\frac{1}{c}\frac{\partial \mathbf{B}^{(i)}}{\partial t}, \quad \operatorname{div} \mathbf{B}^{(i)} = 0, \quad \operatorname{div} \mathbf{D}^{(i)} = 0 \tag{1.3}$$

in the exterior domain (the remaining space, where we have vacuum)

$$\operatorname{rot} \mathbf{H}^{(0)} = \frac{1}{c}\frac{\partial \mathbf{D}^{(e)}}{\partial t}, \ \operatorname{rot} \mathbf{E}^{(e)} = -\frac{1}{c}\frac{\partial \mathbf{B}^{(e)}}{\partial t}, \quad \operatorname{div} \mathbf{B}^{(e)} = 0, \ \operatorname{div} \mathbf{D}^{(e)} = 0 \tag{1.4}$$

Here $\mathbf{E}$ and $\mathbf{D}$ are the intensity and the induction vectors of the electric field, $\mathbf{U}$ (u_α, u_β, u_γ) is the displacement vector of the shell particle and c is the velocity of light in vacuum.

Assuming that the electromagnetic field does not change too fast with time, the relation between the vectors B and H, E and D in the system of coordinates attached to the moving interface of the two media, is taken in the form

$$\mathbf{B}_*^{(i)} = \mu \mathbf{H}_*^{(i)}, \quad \mathbf{B}_*^{(e)} = \mathbf{H}_*^{(e)}, \quad \mathbf{D}_*^{(e)} = \mathbf{E}_*^{(e)}, \quad \mathbf{D}_*^{(i)} = \varepsilon \mathbf{E}_*^{(i)} \tag{1.5}$$

This also means that we do not consider, in particular, shells made of superconducting, ferroelectric and ferromagnetic materials. Obviously, at the interface of the two media we have the following conditions:

$$[\mathbf{B}_*^{(e)} - \mathbf{B}_*^{(i)}] \cdot \mathbf{n}_* = 0, \qquad [\mathbf{H}_*^{(e)} - \mathbf{H}_*^{(i)}] \cdot \mathbf{n}_* = 0 \tag{1.6}$$
$$[\mathbf{D}_*^{(e)} - \mathbf{D}_*^{(i)}] \cdot \mathbf{n}_* = 0, \qquad [\mathbf{E}_*^{(e)} - \mathbf{E}_*^{(i)}] \cdot \mathbf{n}_* = 0$$

Here $\mathbf{n}_*$ is the vector normal to the interface in the moving coordinate system. The vectors which characterize the electromagnetic field under consideration in the moving coordinate system, denoted by an asterisk, can be expressed in terms of the corresponding fixed coordinate system α, β, γ, by the formulas [1, 2]

$$\mathbf{B}_* = \mathbf{B} - \frac{1}{c} \mathbf{v}_n \times \mathbf{E}, \qquad \mathbf{H}_* = \mathbf{H} - \frac{1}{c} \mathbf{v}_n \times \mathbf{D}$$
$$\mathbf{E}_* = \mathbf{E} + \frac{1}{c} \mathbf{v}_n \times \mathbf{B}, \qquad \mathbf{D}_* = \mathbf{D} + \frac{1}{c} \mathbf{v}_n \times \mathbf{H} \tag{1.7}$$

Here $\mathbf{v}_n$ is the velocity vector of the displacement of the interface in the direction of the normal and $\mathbf{n}$ is the vector normal to the interface in the fixed system of coordinates (α, β, γ).

2. As indicated above, we will restrict ourselves to the investigation of magnetoelastic oscillations in the case of small perturbations. Taking for the component of the perturbed electromagnetic field

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \qquad \mathbf{E} = \mathbf{e} \tag{2.1}$$

and taking into account that the components $h_\alpha, h_\beta, h_\gamma$ and $e_\alpha, e_\beta, e_\gamma$ of the induced electromagnetic field are small, we linearize Eqs. (1.3). After some transformations the problem reduces to the simultaneous integration of the following system of linear differential equations (here and in the sequel the indices i are omitted and $\mathbf{k}$ is the unit vector in the direction of the coordinate line γ).

In the interior domain:

electrodynamics equations

$$\operatorname{rot} \mathbf{h} = \frac{4\pi\sigma}{c}\left[\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} \times \mathbf{B}_0\right] + \frac{\varepsilon}{c}\frac{\partial \mathbf{e}}{\partial t} + \frac{\varepsilon\mu - 1}{\mu c^2}\frac{\partial^2 u_\gamma}{\partial t^2}\mathbf{k} \times \mathbf{B}_0$$
$$\operatorname{rot} \mathbf{e} = -\frac{\mu}{c}\frac{\partial \mathbf{h}}{\partial t}, \quad \operatorname{div} \mathbf{h} = 0, \quad \operatorname{div}\left(\mathbf{e} + \frac{\varepsilon\mu - 1}{\varepsilon\mu c}\frac{\partial u_\gamma}{\partial t}\mathbf{k} \times \mathbf{B}_0\right) = 0 \tag{2.2}$$

the equations of motion of an element of the elastic shell

$$H_2 \frac{\partial \sigma_\alpha}{\partial \alpha} + H_1 \frac{\partial \tau_{\alpha\beta}}{\partial \beta} + \frac{1}{H_1}\frac{\partial}{\partial \gamma}(H_1^2 H_2 \tau_{\alpha\gamma}) = \rho \frac{\partial^2 u_\alpha}{\partial t^2} - R_\alpha H_1 H_2 \tag{2.3}$$
$$H_1 \frac{\partial \sigma_\beta}{\partial \beta} + H_2 \frac{\partial \tau_{\alpha\beta}}{\partial \alpha} + \frac{1}{H_2}\frac{\partial}{\partial \gamma}(H_1 H_2^2 \tau_{\beta\gamma}) = \rho \frac{\partial^2 u_\beta}{\partial t^2} - R_\beta H_1 H_2$$
$$H_2 \frac{\partial \tau_{\alpha\gamma}}{\partial \alpha} + H_1 \frac{\partial \tau_{\beta\gamma}}{\partial \beta} + \frac{\partial}{\partial \gamma}(H_1 H_2 \sigma_\gamma) - \sigma_\alpha H_2 \frac{\partial H_1}{\partial \gamma} - \sigma_\beta H_1 \frac{\partial H_2}{\partial \gamma} =$$

$$\rho \frac{\partial^2 u_\gamma}{\partial t^2} - R_\gamma H_1 H_2$$

$$H_1 = A\ (1 + k_1\gamma), \qquad H_2 = B\ (1 + k_2\gamma)$$

Here H_1, H_2 are the Lamé coefficients and $\mathbf{R}$ $(R_\alpha, R_\beta, R_\gamma)$ are the forces of electromagnetic origin which are determined in the following manner [3]:

$$\mathbf{R} = \frac{\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} \times \mathbf{B}_0\right) \times \mathbf{B}_0 \tag{2.4}$$

In the exterior domain:

electrodynamics equations for vacuum

$$\operatorname{rot} \mathbf{h}^{(e)} = \frac{1}{c}\frac{\partial \mathbf{e}^{(e)}}{\partial t}, \quad \operatorname{div} \mathbf{h}^{(e)} = 0 \tag{2.5}$$

$$\operatorname{rot} \mathbf{e}^{(e)} = -\frac{1}{c}\frac{\partial \mathbf{h}^{(e)}}{\partial t}, \qquad \operatorname{div} \mathbf{e}^{(e)} = 0$$

Thus, the problem of the magnetoelastic oscillations of a thin shell has been reduced to the simultaneous integration of the system of equations (2. 2), (2. 3) and (2. 5). Obviously, to these equations we have to adjoin the conditions at the surfaces of the shell for the electromagnetic field, which are obtained from the conditions (1. 7) by linearization

$$h_\gamma = \frac{1}{\mu} h_\gamma^{(e)} + \frac{\mu - 1}{\mu} B_{0\alpha}^{(e)} \frac{1}{A}\frac{\partial u_\gamma}{\partial \alpha} + \frac{\mu - 1}{\mu} B_{0\beta}^{(e)} \frac{1}{B}\frac{\partial u_\gamma}{\partial \beta}$$

$$h_\alpha = h_\alpha^{(e)} + \frac{\mu - 1}{\mu} B_{0\gamma}^{(e)} \frac{1}{A}\frac{\partial u_\gamma}{\partial \alpha}, \quad h_\beta = h_\beta^{(e)} + \frac{\mu - 1}{\mu} B_{0\gamma}^{(e)} \frac{1}{B}\frac{\partial u_\gamma}{\partial \beta} \tag{2.6}$$

$$e_\gamma = \frac{1}{\varepsilon} e_\gamma^{(e)}, \ e_\alpha = e_\alpha^{(e)} + \frac{\mu - 1}{c} B_{0\beta}^{(e)} \frac{\partial u_\gamma}{\partial t}, \ e_\beta = e_\beta^{(e)} - \frac{\mu - 1}{c} B_{0\alpha}^{(e)} \frac{\partial u_\gamma}{\partial t}$$

Finally, we have to adjoin here also the boundary conditions at the ends of the shell and the conditions of damping at infinity of the electromagnetic perturbations.

3. We apply the method of asymptotic integration to the system of equations (2. 2) and (2. 3), restricting ourselves only to the construction of the fundamental iterative process [7 – 9].

As is known, the fundamental iterative process allows us to determine the slowly damped part of the solution, which, for example in the case of the bending problem of a shell, gives the possibility to find in the first approximation that state of stress which characterizes the classical shell theory. Thus, the first approximation of the fundamental iterative process of the three-dimensional problem of the elasticity theory reduces to the two-dimensional problem of the shell theory, constructed according to the hypothesis of non-deformable normals [7, 8].

Following [7 – 9], we assume that the intensity of the electromagnetic field induced in the shell, due to the thinness of the shell, varies slowly with respect to the variables α and β (in the median surface), while with respect to the variable γ (along the thickness of the shell) it varies rapidly.

Expanding the scale with respect to the variable γ. according to the formula

$$\gamma = h\zeta \tag{3.1}$$

and taking into account (1. 5) (the electromagnetic field varies not too rapidly in time) and (3.1), we assume that the rapidity of the variation of the magnetic and electric field intensities induced in the shell is not too large with respect to all four variables α, β, ζ, t.

In the following we will not give the computations related to the asymptotic integration of Eqs. (2.3), since we assume that the results of [8] hold also for the problem under consideration.

Taking into account (3.1), we rewrite the linearized equations (2.2) in the following form:

$$\frac{1}{B}\frac{\partial h_\gamma}{\partial \beta} - h^{-1}\frac{\partial h_\beta}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[e_\alpha + \frac{1}{c}\left(B_{0\gamma}\frac{\partial u_\beta}{\partial t} - B_{0\beta}\frac{\partial u_\gamma}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\alpha}{\partial t} - \frac{\varepsilon\mu - 1}{c^2\mu}B_{0\beta}\frac{\partial^2 u_\gamma}{\partial t^2},$$

$$h^{-1}\frac{\partial h_\alpha}{\partial \zeta} - \frac{1}{A}\frac{\partial h_\gamma}{\partial \alpha} = \frac{4\pi\sigma}{c}\left[e_\beta + \frac{1}{c}\left(B_{0\alpha}\frac{\partial u_\gamma}{\partial t} - B_{0\gamma}\frac{\partial u_\alpha}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\beta}{\partial t} + \frac{\varepsilon\mu - 1}{c^2\mu}B_{0\alpha}\frac{\partial^2 u_\gamma}{\partial t^2}$$

$$\frac{1}{A}\frac{\partial h_\beta}{\partial \alpha} - \frac{1}{B}\frac{\partial h_\alpha}{\partial \beta} = \frac{4\pi\sigma}{c}\left[e_\gamma + \frac{1}{c}\left(B_{0\beta}\frac{\partial u_\beta}{\partial t} - B_{0\alpha}\frac{\partial u_\beta}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\gamma}{\partial t} \tag{3.2}$$

$$\frac{1}{A}\frac{\partial e_\alpha}{\partial \alpha} + \frac{1}{B}\frac{\partial e_\beta}{\partial \beta} + h^{-1}\frac{\partial e_\gamma}{\partial \zeta} + \frac{\varepsilon\mu - 1}{\varepsilon\mu c}\left(\frac{B_{0\alpha}}{B}\frac{\partial^2 u_\gamma}{\partial\beta\partial t} - \frac{B_{0\beta}}{A}\frac{\partial^2 u_\gamma}{\partial\alpha\partial t}\right) = 0$$

$$\frac{1}{A}\frac{\partial e_\beta}{\partial \alpha} - \frac{1}{B}\frac{\partial e_\alpha}{\partial \beta} = -\frac{\mu}{c}\frac{\partial h_\gamma}{\partial t}, \qquad \frac{1}{B}\frac{\partial e_\gamma}{\partial \beta} - h^{-1}\frac{\partial e_\beta}{\partial \zeta} = -\frac{\mu}{c}\frac{\partial h_\alpha}{\partial t}$$

$$h^{-1}\frac{\partial e_\alpha}{\partial \zeta} - \frac{1}{A}\frac{\partial e_\gamma}{\partial \alpha} = -\frac{\mu}{c}\frac{\partial h_\beta}{\partial t}, \qquad \frac{1}{A}\frac{\partial h_\alpha}{\partial \alpha} + \frac{1}{B}\frac{\partial h_\beta}{\partial \beta} + h^{-1}\frac{\partial h_\gamma}{\partial \zeta} = 0 \tag{3.3}$$

We write an arbitrary component of the electromagnetic field or of the displacement of the shell in the form

$$Q = h^{-q}\sum_{s=1}^{S} h^{s-1}Q^{(s)} \tag{3.4}$$

where q is an integer, different for different components of the electromagnetic field and of the shell displacement. This has to be chosen in such a way that, after inserting (3.4) into Eqs. (3.2) and (3.3) and making equal to zero in each equation the coefficients of the same powers of h, we should obtain a consistent sequence of systems of equations for the determination of the coefficients of the expansions (3.4). Numerous solutions of the classical problems of shell theory, without considering electromagnetic effects, show that in the representations of the displacements with the formula (3.4), the exponent q can be selected in the following manner [8]:

$$(u_\alpha, u_\beta, u_\gamma) \to q = a \tag{3.5}$$

As far as the components of the induced electromagnetic field are concerned, the exponent q is chosen in the following manner:

$$(h_\alpha, h_\beta, e_\gamma) \to q = b, \qquad (h_\gamma, e_\alpha, e_\beta) \to q = b + 1 \tag{3.6}$$

Considering other variants of the exponent q for the components of the electromagnetic field in the representation (3.4), we obtain a contradiction.

For convenience we introduce a new notation $k = b - a + 1$. Inserting into Eqs. (3.3) the values of the components of the electromagnetic field of the shell from (3.4), taking into account (3.6) and equating the coefficients of the same powers of h in each equation separately, we obtain the following unique (independent of k) system of equations:

$$\frac{\partial e_\beta^{(s)}}{\partial \zeta} = \frac{1}{B}\frac{\partial e_\gamma^{(s-2)}}{\partial \beta} + \frac{\mu}{c}\frac{\partial h_\alpha^{(s-2)}}{\partial t}, \qquad \frac{\partial e_\alpha^{(s)}}{\partial \zeta} = \frac{1}{A}\frac{\partial e_\gamma^{(s-2)}}{\partial \alpha} - \frac{\mu}{c}\frac{\partial h_\beta^{(s-2)}}{\partial t}$$

$$\frac{1}{A}\frac{\partial e_\beta^{(s)}}{\partial \alpha} - \frac{1}{B}\frac{\partial e_\alpha^{(s)}}{\partial \beta} = -\frac{\mu}{c}\frac{\partial h_\gamma^{(s)}}{\partial t}, \qquad \frac{\partial h_\gamma^{(s)}}{\partial \zeta} - \frac{1}{A}\frac{\partial h_\alpha^{(s-2)}}{\partial \alpha} - \frac{1}{B}\frac{\partial h_\beta^{(s-2)}}{\partial \beta} \tag{3.7}$$

Equations (3.2), after similar transformations, taking into account of (3.5) and (3.6), lead to the mutually distinct (for each value of k) systems of equations

$$\frac{1}{B}\frac{\partial h_\gamma^{(s)}}{\partial \beta} - \frac{\partial h_\beta^{(s)}}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[e_\alpha^{(s)} + \frac{1}{c}\left(B_{0\gamma}\frac{\partial u_\beta^{(s-k)}}{\partial t} - B_{0\beta}\frac{\partial u_\gamma^{(s-k)}}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\alpha^{(s)}}{\partial t} - \frac{\varepsilon\mu - 1}{c^2\mu}B_{0\beta}\frac{\partial^2 u_\gamma^{(s-k)}}{\partial t^2}$$

$$\frac{\partial h_\alpha^{(s)}}{\partial \zeta} - \frac{1}{A}\frac{\partial h_\gamma^{(s)}}{\partial \alpha} = \frac{4\pi\sigma}{c}\left[e_\beta^{(s)} + \frac{1}{c}\left(B_{0\alpha}\frac{\partial u_\gamma^{(s-k)}}{\partial t} - B_{0\gamma}\frac{\partial u_\alpha^{(s-k)}}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\beta^{(s)}}{\partial t} - + \frac{\varepsilon\mu - 1}{c^2\mu}B_{0\alpha}\frac{\partial^2 u_\gamma^{(s-k)}}{\partial t^2}$$

$$\frac{1}{A}\frac{\partial h_\beta^{(s)}}{\partial \alpha} - \frac{1}{B}\frac{\partial h_\alpha^{(s)}}{\partial \beta} = \frac{4\pi\sigma}{c}\left[e_\gamma^{(s)} + \frac{1}{c}\left(B_{0\beta}\frac{\partial u_\alpha^{(s-k+1)}}{\partial t} - B_{0\alpha}\frac{\partial u_\beta^{(s-k+1)}}{\partial t}\right)\right] + \frac{\varepsilon}{c}\frac{\partial e_\gamma^{(s)}}{\partial t} \tag{3.8}$$

$$\frac{1}{A}\frac{\partial e_\alpha^{(s)}}{\partial \alpha} + \frac{1}{B}\frac{\partial e_\beta^{(s)}}{\partial \beta} + \frac{\partial e_\gamma^{(s)}}{\partial \zeta} + \frac{\varepsilon\mu - 1}{\varepsilon\mu c}\left(\frac{B_{0\alpha}}{B}\frac{\partial^2 u_\gamma^{(s-k)}}{\partial \beta \partial t} - \frac{B_{0\beta}}{A}\frac{\partial^2 u_\gamma^{(s-k)}}{\partial \alpha \partial t}\right) = 0$$

Equations (3.7) combined with (3.8) form, separately for every value of k a chain of systems of equations of the fundamental iterative process, resulting in the successive $(s=1, 2, 3, \ldots)$ determination of the unknowns $Q^{(s)}$. In this connection we have to assume that $Q^{(s)} \equiv 0$ for $s < 1$, and also, that in the construction of $Q^{(s+1)}$ the corresponding quantities $Q^{(1)}, Q^{(2)}, \ldots, Q^{(s)}$ are considered known.

Let us consider different values of the number k. The case $k < 0$ does not present any interest, since in this case, according to (3.8), either all the components of the given (unperturbed) magnetic field must be equal to zero or we arrive at a contradiction, consisting in the superposition of additional constraints on the elastic displacements. In the case $k = 0$, from Eqs. (3.8) we obtain in the first approximation of the asymptotic integration $(s = 1)$

$$B_{0\beta}\frac{\partial u_\alpha^{(1)}}{\partial t} - B_{0\alpha}\frac{\partial u_\beta^{(1)}}{\partial t} = 0 \tag{3.9}$$

Thus, the case $k = 0$ is possible by satisfying the condition (3.9), which depends both on the velocity of the oscillations and on the magnitude of the given magnetic field. For the nonstationary problem, this is possible in the case when $B_{0\alpha} = B_{0\beta} = 0$, i.e. when the given magnetic field is perpendicular to the median surface of the shell, otherwise one obtains an additional restriction on the elastic displacements. In this connection, according to (1.2), we note that the given magnetic field must be constant in the coordinate system α, β, γ. Taking $B_{0\alpha} = B_{0\beta} = 0$, from (3.8) we obtain the following equations in the case $k = 0$:

$$\begin{gathered}
\frac{1}{B}\frac{\partial h_\gamma^{(s)}}{\partial\beta} - \frac{\partial h_\beta^{(s)}}{\partial\zeta} = \frac{4\pi\sigma}{c}\left[e_\alpha^{(s)} + \frac{1}{c}B_{0\gamma}\frac{\partial u_\beta^{(s)}}{\partial t}\right] + \frac{\varepsilon}{c}\frac{\partial e_\alpha^{(s)}}{\partial t} \\
\frac{\partial h_\alpha^{(s)}}{\partial\zeta} - \frac{1}{A}\frac{\partial h_\gamma^{(s)}}{\partial\alpha} = \frac{4\pi\sigma}{c}\left[e_\beta^{(s)} - \frac{1}{c}B_{0\gamma}\frac{\partial u_\alpha^{(s)}}{\partial t}\right] + \frac{\varepsilon}{c}\frac{\partial e_\beta^{(s)}}{\partial t} \\
\frac{1}{A}\frac{\partial h_\beta^{(s)}}{\partial\alpha} - \frac{1}{B}\frac{\partial h_\alpha^{(s)}}{\partial\beta} = \frac{4\pi\sigma}{c}e_\gamma^{(s)} + \frac{\varepsilon}{c}\frac{\partial e_\gamma^{(s)}}{\partial t} \\
\frac{1}{A}\frac{\partial e_\alpha^{(s)}}{\partial\alpha} + \frac{1}{B}\frac{\partial e_\beta^{(s)}}{\partial\beta} + \frac{\partial e_\gamma^{(s)}}{\partial\zeta} = 0
\end{gathered} \tag{3.10}$$

From here it is clear that for $k = 0$, in the first approximation, the electrodynamics equations do not separate from the equations of motion of the elastic shell.

In the case $k = 1$, Eqs. (3.8) show that even in the first approximation $(s = 1)$, independent of the given magnetic field, the electrodynamics equations do not separate from equations of motion of the shell.

It is easy to see that for $k > 1$ (the exponent of the intensity of the electromagnetic field is significantly larger than the exponent of the intensity of the shell displacements), in the first approximation, the components of the electromagnetic field can be determined independently from the elastic oscillations of the shell.

We note that in the case $k < 0$, the exponent of the intensity of the electromagnetic field is significantly smaller than the exponent of the intensity of the shell displacements, while in the cases $k = 0$, $k = 1$ they differ little from each other.

We represent the solution of the system of equations (3.7) and (3.8) in the form of a sum of two terms: $Q^{(s)} = Q_i^{(s)} + Q^{*(s)}$. The first term denotes the solution of the homogeneous system obtained by discarding the quantities whose superscript is less than s, while the second term denotes some particular solution of the indicated system in which all the quantities whose superscript is less than s are considered to be known.

Let us consider the system of homogeneous equations. For all $k \geqslant 0$, those equations of the homogeneous system which are obtained from all the equations of the system (3.7) and the last equations of the systems (3.8) and (3.10), are common and have the form

$$\frac{\partial e_\alpha^{(s)}}{\partial\zeta} = 0, \quad \frac{\partial e_\beta^{(s)}}{\partial\zeta} = 0, \quad \frac{\partial h_\gamma^{(s)}}{\partial\zeta} = 0 \tag{3.11}$$

$$\frac{1}{A}\frac{\partial e_\beta^{(s)}}{\partial\alpha} - \frac{1}{B}\frac{\partial e_\alpha^{(s)}}{\partial\beta} = -\frac{\mu}{c}\frac{\partial h_\gamma^{(s)}}{\partial t}, \quad \frac{1}{A}\frac{\partial e_\alpha^{(s)}}{\partial\alpha} + \frac{1}{B}\frac{\partial e_\beta^{(s)}}{\partial\beta} + \frac{\partial e_\gamma^{(s)}}{\partial\zeta} = 0$$

The remaining equations of the homogeneous system for distinct values of k are distinct and are obtained from the remaining equations of the corresponding systems (3.8) and (3.10) in the form

$$\operatorname{rot}\mathbf{h}^{(s)} = \frac{4\pi\sigma}{c}\left[\mathbf{e}^{(s)} + \frac{B_{0\gamma}}{c}\frac{\partial \mathbf{U}^{(s)}}{\partial t}\times\mathbf{k}\right] + \frac{\varepsilon}{c}\frac{\partial \mathbf{e}^{(s)}}{\partial t} \qquad (k=0) \qquad (3.12)$$

$$\operatorname{rot}\mathbf{h}^{(s)} = \frac{4\pi\sigma}{c}\left[\mathbf{e}^{(s)} + \frac{\delta_{is}}{c}\frac{\partial \mathbf{U}^{(s)}}{\partial t}\times\mathbf{B}_0\right] + \frac{\varepsilon}{c}\frac{\partial \mathbf{e}^{(s)}}{\partial t} \qquad (k=1) \qquad (3.13)$$

$$\operatorname{rot}\mathbf{h}^{(s)} = \frac{4\pi\sigma}{c}\mathbf{e}^{(s)} + \frac{\varepsilon}{c}\frac{\partial \mathbf{e}^{(s)}}{\partial t} \qquad (k>1) \qquad (3.14)$$

The system of homogeneous equations obtained in this way can be easily integrated for each value of k. In this case, independent of k, according to (3.11), $e_{\alpha i}^{(s)}$, $e_{\beta i}^{(s)}$, $e_{\gamma i}^{(s)}$, $h_{\gamma i}^{(s)}$ have the following form:

$$e_{\alpha i}^{(s)} = e_{\alpha 0}^{(s)}(\alpha,\beta,t),\ e_{\beta i}^{(s)} = e_{\beta 0}^{(s)}(\alpha,\beta,t),\ h_{\gamma i}^{(s)} = h_{\gamma 0}^{(s)}(\alpha,\beta,t) \qquad (3.15)$$

$$e_{\gamma i}^{(s)} = -\zeta\left[\frac{1}{A}\frac{\partial e_{\alpha 0}^{(s)}}{\partial\alpha} + \frac{1}{B}\frac{\partial e_{\beta 0}^{(s)}}{\partial\beta}\right]$$

According to what we have said at the beginning of Sect. 3, we also give the corresponding expressions for the displacements [8]

$$u_{\gamma i}^{(s)} = u_{\gamma 0}^{(s)}(\alpha,\beta,t) = w_0^{(s)}(\alpha,\beta,t) \qquad (3.16)$$

$$u_{\alpha i}^{(s)} = u_0^{(s)}(\alpha,\beta,t) - \frac{\zeta}{A}\frac{\partial w_0^{(s)}}{\partial\alpha}, \qquad u_{\beta i}^{(s)} = v_0^{(s)}(\alpha,\beta,t) - \frac{\zeta}{B}\frac{\partial w_0^{(s)}}{\partial\beta}$$

Now, making use of (3.15) and (3.16), from Eqs. (3.12)–(3.14) we can determine $h_{\alpha i}^{(s)}$, $h_{\beta i}^{(s)}$, distinct for different values of k

$$h_{\alpha i}^{(s)} = \zeta\left[\frac{1}{A}\frac{\partial h_{\gamma 0}^{(s)}}{\partial\alpha} + \frac{4\pi\sigma}{c}\left(e_{\beta 0}^{(s)} - \frac{B_{0\gamma}}{c}\frac{\partial u_0^{(s)}}{\partial t} + \frac{B_{0\gamma}}{c}\frac{\zeta}{2A}\frac{\partial^2 w_0^{(s)}}{\partial\alpha\partial t}\right) + \frac{\varepsilon}{c}\frac{\partial e_{\beta 0}^{(s)}}{\partial t}\right]$$

$$h_{\beta i}^{(s)} = \zeta\left[\frac{1}{B}\frac{\partial h_{\gamma 0}^{(s)}}{\partial\beta} - \frac{4\pi\sigma}{c}\left(e_{\alpha 0}^{(s)} + \frac{B_{0\gamma}}{c}\frac{\partial v_0^{(s)}}{\partial t} - \frac{B_{0\gamma}}{c}\frac{\zeta}{2B}\frac{\partial^2 w_0^{(s)}}{\partial\beta\partial t}\right) + \frac{\varepsilon}{c}\frac{\partial e_{\alpha 0}^{(s)}}{\partial t}\right]$$

$$(k=0) \qquad\qquad (k=0)$$

$$h_{\alpha i}^{(s)} = \zeta\left[\frac{1}{A}\frac{\partial h_{\gamma 0}^{(s)}}{\partial\alpha} + \frac{4\pi\sigma}{c}e_{\beta 0}^{(s)} + \frac{\varepsilon}{c}\frac{\partial e_{\beta 0}^{(s)}}{\partial t}\right] \qquad (3.17)$$

$$h_{\beta i}^{(s)} = \zeta\left[\frac{1}{B}\frac{\partial h_{\gamma 0}^{(s)}}{\partial\beta} - \frac{4\pi\sigma}{c}e_{\alpha 0}^{(s)} - \frac{\varepsilon}{c}\frac{\partial e_{\alpha 0}^{(s)}}{\partial t}\right] \qquad (k\geqslant 1) \qquad (3.18)$$

Let us find particular solutions. The expressions $e_\alpha^{*(s)}, e_\beta^{*(s)}, h_\gamma^{*(s)}$ are determined from (3.7) independent of k

$$e_\alpha^{*(s)} = \int_0^\zeta\left[\frac{1}{A}\frac{\partial e_\gamma^{(s-2)}}{\partial\alpha} - \frac{\mu}{c}\frac{\partial h_\beta^{(s-2)}}{\partial t}\right]d\zeta,\quad e_\beta^{*(s)} = \int_0^\zeta\left[\frac{1}{B}\frac{\partial e_\gamma^{(s-2)}}{\partial\beta} + \frac{\mu}{c}\frac{\partial h_\alpha^{(s-2)}}{\partial t}\right]d\zeta$$

$$h_\gamma^{*(s)} = -\int_0^\zeta\left[\frac{1}{A}\frac{\partial h_\alpha^{(s-2)}}{\partial\alpha} + \frac{1}{B}\frac{\partial h_\beta^{(s-2)}}{\partial\beta}\right]d\zeta \qquad (3.19)$$

The remaining quantities are determined from the formulas

$$h_\alpha^{*(s)} = \int_0^\zeta \left\{ \frac{1}{} \frac{\partial h_\gamma^{*(s)}}{\partial \alpha} + \frac{4\pi\sigma}{c} \left[e_\beta^{*(s)} + \frac{1}{c} \left(B_{0\alpha} \frac{\partial u_\gamma^{(s-k)}}{\partial t} - B_{0\gamma} \frac{\partial u_\alpha^{(s-k)}}{\partial t} \right) \right] + \right.$$
$$\left. \frac{\varepsilon}{c} \frac{\partial e_\beta^{*(s)}}{\partial t} + \frac{\varepsilon\mu - 1}{c^2\mu} B_{0\alpha} \frac{\partial^2 u_\gamma^{(s-k)}}{\partial t^2} \right\} d\zeta \tag{3.20}$$
$$h_\beta^{*(s)} = \int_0^\zeta \left\{ \frac{1}{B} \frac{\partial h_\gamma^{*(s)}}{\partial \beta} - \frac{4\pi\sigma}{c} \left[e_\alpha{}^{*} + \frac{1}{c} \left(B_{0\gamma} \frac{\partial u_\beta^{(s-k)}}{\partial t} - B_{0\beta} \frac{\partial u_\gamma^{(s-k)}}{\partial t} \right) \right] - \right.$$
$$\left. \frac{\varepsilon}{c} \frac{\partial e_\alpha^{*(s)}}{\partial t} + \frac{\varepsilon\mu - 1}{c^2\mu} B_{0\beta} \frac{\partial^2 u_\gamma^{(s-k)}}{\partial t^2} \right\} d\zeta$$
$$e_\gamma^{*(s)} = -\int_0^\zeta \left[\frac{1}{A} \frac{\partial e_\alpha^{*(s)}}{\partial \alpha} + \frac{1}{B} \frac{\partial e_\beta^{*(s)}}{\partial \beta} + \frac{\varepsilon\mu - 1}{\varepsilon\mu c} \left(\frac{B_{0\alpha}}{B} \frac{\partial^2 u_\gamma^{(s-k)}}{\partial \beta \partial t} - \frac{B_{0\beta}}{A} \frac{\partial^2 u_\gamma^{(s-k)}}{\partial \alpha \partial t} \right) \right] d\zeta$$

where in the case $k = 0$ it is necessary to take $B_{0\alpha} = B_{0\beta} = 0$.

In the formulas (3.19), (3.20), the quantities marked with an asterisk are functions of the variables α, β, ζ, t; the quantities not marked with an asterisk and having index less than s are assumed to be known.

As indicated above, $Q^{(s)} \equiv 0$ for $s < 1$. Therefore from (3.19) it follows that $Q^{*(1)}$ and $Q^{*(2)}$ are identically equal to zero. (Obviously, the same is the case in the representation of the displacements u_α, u_β, u_γ [7 – 9]).

Examining the obtained solutions (3.15) and (3.19) of the linearized magnetoelasticity equations, we note that in the case that the hypothesis of the nondeformable normals holds (3.16), the components e_α, e_β and h_γ of the induced electromagnetic field, up to the third approximation of the asymptotic integration, do not depend on the coordinate ζ.

Thus, similar to the classical theory of thin shells [4 – 6], we can formulate the following fundamental hypotheses for the magnetoelasticity of a thin shell (assumptions (a) and (b) taken from the classical theory of shells, are given here for the sake of completeness):

a) the normal to the median surface of a rectilinear element of the shell remains, after deformation, rectilinear and normal to the deformed median surface of the shell and preserves its length;

b) the normal stress σ_γ can be neglected in comparison with the other stresses;

c) the tangential components of the intensity vector of the induced electric field and the normal component of the intensity vector of the induced magnetic field do not vary along the thickness of the shell.

Obviously, all the three assumptions have to be considered as individual parts of a unique hypothesis, on the basis of which the three-dimensional problem of the magnetoelasticity of a continuous body reduces to the two-dimensional magnetoelasticity problem of a thin shell.

4. The above formulated fundamental hypotheses for the interior problem can be written analytically in the following manner:

$$u_\alpha = u - \frac{\gamma}{A} \frac{\partial w}{\partial \alpha}, \qquad u_\beta = v - \frac{\gamma}{B} \frac{\partial w}{\partial \beta}, \qquad u_\gamma = w \tag{4.1}$$
$$e_\alpha = \varphi, \qquad e_\beta = \psi, \qquad h_\gamma = f$$

Here u v, w are the desired displacements of the median surface of the shell; φ, ψ, f are the desired functions of the excited electromagnetic field (all quantities are functions of α, β, t).

The equations of motion (2.3) of the electrically conducting shell can be written in terms of forces and moments in the form

$$\frac{1}{A}\frac{\partial T_\alpha}{\partial \alpha}+\frac{1}{B}\frac{\partial T_{\alpha\beta}}{\partial \beta}=2\rho h\frac{\partial^2 u}{\partial t^2}-X_\alpha \tag{4.2}$$

$$\frac{1}{B}\frac{\partial T_\beta}{\partial \beta}+\frac{1}{A}\frac{\partial T_{\alpha\beta}}{\partial \alpha}=2\rho h\frac{\partial^2 v}{\partial t^2}-X_\beta$$

$$\frac{1}{A^2}\frac{\partial^2 M_\alpha}{\partial \alpha^2}+\frac{2}{AB}\frac{\partial^2 M_{\alpha\beta}}{\partial\alpha\partial\beta}+\frac{1}{B^2}\frac{\partial^2 M_\beta}{\partial \beta^2}+2\rho h\frac{\partial^2 w}{\partial t^2}-$$

$$(k_1T_\alpha+k_2T_\beta)=P-X_\gamma-\frac{1}{A}\frac{\partial m_\alpha}{\partial\alpha}-\frac{1}{B}\frac{\partial m_\beta}{\partial\beta}$$

Here T_α, T_β, $T_{\alpha\beta}$, M_α, M_β, $M_{\alpha\beta}$ are the interior forces and moments, which in terms of the displacements of the median surface can be represented by the usual elasticity relations [4 – 6]

$$T_\alpha=\frac{2Eh}{1-\nu^2}\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}+k_1w\right)+\frac{2\nu Eh}{1-\nu^2}\left(\frac{1}{B}\frac{\partial v}{\partial\beta}+k_2w\right)$$

$$T_\beta=\frac{2Eh}{1-\nu^2}\left(\frac{1}{B}\frac{\partial v}{\partial\beta}+k_2w\right)+\frac{2\nu Eh}{1-\nu^2}\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}+k_1w\right) \tag{4.3}$$

$$T_{\alpha\beta}=\frac{Eh}{1+\nu}\left(\frac{1}{A}\frac{\partial v}{\partial\alpha}+\frac{1}{B}\frac{\partial u}{\partial\beta}\right),\qquad M_{\alpha\beta}=-\frac{2Eh^3}{3(1+\nu)}\frac{1}{AB}\frac{\partial^2 w}{\partial\alpha\partial\beta}$$

$$M_\alpha=-\frac{2Eh^3}{3(1-\nu^2)}\left(\frac{1}{A^2}\frac{\partial^2 w}{\partial\alpha^2}+\frac{\nu}{B^2}\frac{\partial^2 w}{\partial\beta^2}\right),$$

$$M_\beta=-\frac{2Eh^3}{3(1-\nu^2)}\left(\frac{1}{B^2}\frac{\partial^2 w}{\partial\beta^2}+\frac{\nu}{A^2}\frac{\partial^2 w}{\partial\alpha^2}\right)$$

where P is the normal component of the exterior surface load, X $(X_\alpha, X_\beta, X_\gamma)$, m $(m_\alpha, m_\beta, m_\gamma)$ are the forces and moments of electromagnetic origin which according to (2.4) are determined in the following manner:

$$\mathrm{X}=\int_{-h}^{h}R\,d\gamma,\qquad \mathrm{m}=\int_{-h}^{h}R\gamma\,d\gamma \tag{4.4}$$

For the remaining components h_α, h_β, e_γ of the electromagnetic field inside the shell, we obtain from Eqs. (2.2) by integration with respect to γ from zero to γ and by taking into account (4.1) and the surface conditions (2.6)

$$h_\alpha=\frac{h_\alpha^{+}+h_\alpha^{-}}{2}+\gamma\left(\frac{1}{A}\frac{\partial f}{\partial\alpha}+\frac{\varepsilon}{c}\frac{\partial\psi}{\partial t}+\frac{4\pi\sigma}{c}\psi\right)+$$

$$\frac{4\pi\sigma}{c^2}\left(a_\alpha\frac{\partial w}{\partial t}+\frac{a}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}-a_\gamma\frac{\partial u}{\partial t}\right)+\frac{\varepsilon\mu-1}{c^2\mu}a_\alpha\frac{\partial^2 w}{\partial t^2}$$

$$h_\beta=\frac{h_\beta^{+}+h_\beta^{-}}{2}+\gamma\left(\frac{1}{B}\frac{\partial f}{\partial\beta}-\frac{\varepsilon}{c}\frac{\partial\varphi}{\partial t}-\frac{4\pi\sigma}{c}\varphi\right)+ \tag{4.5}$$

$$\frac{4\pi\sigma}{c^2}\left(a_\beta\frac{\partial w}{\partial t}+\frac{a}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}-a_\gamma\frac{\partial v}{\partial t}\right)+\frac{\varepsilon\mu-1}{c^2\mu}a_\beta\frac{\partial^2 w}{\partial t^2}$$

$$e_\gamma = \frac{e_\gamma^+ + e_\gamma^-}{2} - \gamma\left(\frac{1}{A}\frac{\partial\varphi}{\partial\alpha} + \frac{1}{B}\frac{\partial\psi}{\partial\beta}\right) + \frac{\varepsilon\mu - 1}{\varepsilon\mu c}\left(a_\beta \frac{1}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t} - a_\alpha \frac{1}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right)$$

$$a_i = \int_0^\gamma B_{0i}d\gamma - \frac{1}{2}\left(\int_0^h B_{0i}d\gamma + \int_0^{-h} B_{0i}d\gamma\right) \quad (i = \alpha, \beta, \gamma)$$

$$a = \int_0^\gamma \gamma B_{0\gamma}d\gamma - \frac{1}{2}\left(\int_0^h \gamma B_{0\gamma}d\gamma + \int_0^{-h} \gamma B_{0\gamma}d\gamma\right)$$

The plus and minus superscripts indicate the values of the corresponding quantities for $\gamma = h$ and $\gamma = -h$.

Thus, all the components of the electromagnetic field of the shell, the forces and the moments which occur in the magnetoelastic equations of the shell, can be represented by six unknown functions $(u, v, w, \varphi, \psi, f)$ and by the values of the components h_α, h_β, e_γ of the electromagnetic field at the surfaces of the shell.

If we restrict ourselves to the investigation of the proper problems of the mechanics of an elastic shell, then, as we will see, the fundamental resolvent equations will contain only the unknown function of the values of the normal component of the electric field at the surfaces of the shell $(\gamma = \pm h)$.

The values of e_γ at the surfaces of the shell can be represented in terms of the unknown functions in the following manner:

$$\begin{aligned} e_\gamma^+ &= -\frac{1}{c}\left[B_{0\beta}^+\frac{\partial u}{\partial t} - B_{0\beta}^+\frac{h}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t} - B_{0\alpha}^+\frac{\partial v}{\partial t} + B_{0\alpha}^+\frac{h}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right] \\ e_\gamma^- &= -\frac{1}{c}\left[B_{0\beta}^-\frac{\partial u}{\partial t} + B_{0\beta}^-\frac{h}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t} - B_{0\alpha}^-\frac{\partial v}{\partial t} - B_{0\alpha}^-\frac{h}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right] \end{aligned} \quad (4.6)$$

Formulas (4.6) are obtained from the condition of the vanishing of the normal component of the current density on the surfaces $\gamma = \pm h$, i.e. from the condition (since the shell is in vacuum)

$$\left[\mathbf{e} + \frac{1}{c}\left(\frac{\partial\mathbf{U}}{\partial t} \times \mathbf{B}_0\right)\right]\cdot\mathbf{n} = 0 \quad (4.7)$$

5. It remains to write out the resolving equations relative to the six unknown functions of the problem. They can be obtained from the consideration of the simultaneous equations of magnetoelasticity.

Substituting into the motion equations (4.2) of the shell the values of the interior forces and moments from (4.3), as well as the forces and moments of electromagnetic origin from (4.4), we obtain, taking into account (4.5) and (4.6)

$$\frac{1}{A^2}\frac{\partial^2 u}{\partial\alpha^2} + \frac{1-\nu}{2B^2}\frac{\partial^2 u}{\partial\beta^2} + \frac{1+\nu}{2AB}\frac{\partial^2 v}{\partial\alpha\,\partial\beta} + \frac{k_1 + \nu k_2}{A}\frac{\partial w}{\partial\alpha} + \frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\Big[b_\gamma\psi +$$
$$c_\beta\left(\frac{1}{A}\frac{\partial\varphi}{\partial\alpha} + \frac{1}{B}\frac{\partial\psi}{\partial\beta}\right) + \frac{1}{c}\Big(F_{\beta\beta}\frac{\partial u}{\partial t} - F_{\alpha\beta}\frac{\partial v}{\partial t} +$$
$$b_{\alpha\gamma}\frac{\partial w}{\partial t} + G_{\beta\beta}\frac{1}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t} - G_{\alpha\beta}\frac{1}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\Big)\Big] = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2}$$

$$\frac{1}{B^2}\frac{\partial^2 v}{\partial\beta^2} + \frac{1-\nu}{2A^2}\frac{\partial^2 v}{\partial\alpha^2} + \frac{1+\nu}{2AB}\frac{\partial^2 u}{\partial\alpha\,\partial\beta} + \frac{k_2 + \nu k_1}{B}\frac{\partial w}{\partial\beta} + \frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\Big[-b_\gamma\varphi -$$
$$c_\alpha\left(\frac{1}{A}\frac{\partial\varphi}{\partial\alpha} + \frac{1}{B}\frac{\partial\psi}{\partial\beta}\right) + \frac{1}{c}\Big(F_{\alpha\alpha}\frac{\partial v}{\partial t} - F_{\beta\alpha}\frac{\partial u}{\partial t} + b_{\beta\gamma}\frac{\partial w}{\partial t} +$$

$$G_{\beta\alpha}\frac{1}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}-G_{\alpha\alpha}\frac{1}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\Big)\Big]=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}$$

$$D\left\{\frac{1}{A^4}\frac{\partial^4 w}{\partial\alpha^4}+\frac{2}{A^2B^2}\frac{\partial^4 w}{\partial\alpha^2\partial\beta^2}+\frac{1}{B^4}\frac{\partial^4 w}{\partial\beta^4}+\frac{3}{h_2}\Big[\frac{k_1+\nu k_2}{A}\frac{\partial u}{\partial\alpha}+\frac{k_2+\nu k_1}{B}\frac{\partial v}{\partial\beta}+\right.$$
$$\left.(k_1^2+k_2^2+2\nu k_1k_2)\,w\Big]\right\}+2\rho h\frac{\partial^2 w}{\partial t^2}=P+\frac{\sigma}{c}\left\{\left(\frac{1}{A}\frac{\partial c_\gamma}{\partial\alpha}-b_\alpha\right)\psi+\right.$$
$$\left(b_\beta-\frac{1}{B}\frac{\partial c_\gamma}{\partial\beta}\right)\varphi+c_\gamma\left(\frac{1}{A}\frac{\partial\psi}{\partial\alpha}-\frac{1}{B}\frac{\partial\varphi}{\partial\beta}\right)+\frac{g_\beta}{A^2}\frac{\partial^2\varphi}{\partial\alpha^2}-\frac{g_\alpha}{AB}\frac{\partial^2\varphi}{\partial\alpha\,\partial\beta}+$$

$$\frac{g_\beta}{AB}\frac{\partial^2\psi}{\partial\alpha\,\partial\beta}-\frac{g_\alpha}{B^2}\frac{\partial^2\psi}{\partial\beta^2}+\frac{1}{c}\Big[\left(b_{\alpha\gamma}+\frac{1}{A}\frac{\partial L_{\beta\beta}}{\partial\alpha}-\frac{1}{B}\frac{\partial L_{\beta\alpha}}{\partial\beta}\right)\frac{\partial u}{\partial t}+\left(b_{\beta\gamma}-\frac{1}{A}\frac{\partial L_{\alpha\beta}}{\partial\alpha}+\right.$$
$$\left.\frac{1}{B}\frac{\partial L_{\alpha\alpha}}{\partial\beta}\right)\frac{\partial v}{\partial t}-\left(b_{\alpha\alpha}+b_{\beta\beta}-\frac{1}{A}\frac{\partial c_{\alpha\gamma}}{\partial\alpha}+\frac{1}{B}\frac{\partial c_{\beta\gamma}}{\partial\beta}\right)\frac{\partial w}{\partial t}-$$
$$\left(\frac{1}{A}\frac{\partial N_{\beta\beta}}{\partial\alpha}-\frac{1}{B}\frac{\partial N_{\beta\alpha}}{\partial\beta}\right)\frac{1}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}+\left(\frac{1}{A}\frac{\partial N_{\alpha\beta}}{\partial\alpha}-\frac{1}{B}\frac{\partial N_{\alpha\alpha}}{\partial\beta}\right)\frac{1}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}+$$
$$\frac{L_{\beta\beta}}{A}\frac{\partial^2 u}{\partial\alpha\,\partial t}-\frac{L_{\beta\alpha}}{B}\frac{\partial^2 u}{\partial\beta\,\partial t}-\frac{L_{\alpha\beta}}{A}\frac{\partial^2 v}{\partial\alpha\partial t}+\frac{L_{\alpha\alpha}}{B}\frac{\partial^2 v}{\partial\beta\,\partial t}-\frac{N_{\beta\beta}}{A^2}\frac{\partial^3 w}{\partial\alpha^2\partial t}- \tag{5.1}$$
$$\left.\frac{N_{\alpha\alpha}}{B^2}\frac{\partial^3 w}{\partial\beta^2\partial t}+\frac{1}{AB}(N_{\alpha\beta}+N_{\beta\alpha})\frac{\partial^3 w}{\partial\alpha\partial\beta\partial t}\Big]\right\}$$

Here

$$b_i=\int_{-h}^{h}B_{0i}d\gamma,\qquad c_i=\int_{-h}^{h}\gamma B_{0i}d\gamma,\qquad b_{ij}=\int_{-h}^{h}B_{0i}B_{0j}d\gamma$$

$$c_{ij}=\int_{-h}^{h}\gamma B_{0i}B_{0j}d\gamma,\quad d_{ij}=\int_{-h}^{h}a_iB_{0j}d\gamma,\quad g_i=\int_{-h}^{h}\gamma^2B_{0i}d\gamma$$

$$g_{ij}=\int_{-h}^{h}\gamma^2B_{0i}B_{0j}d\gamma,\qquad a_{ij}=\int_{-h}^{h}\gamma a_iB_{0j}d\gamma$$

$$F_{ij}=C_i^{+}b_j-b_{ij}-\delta_{ij}b_{\gamma\gamma},\qquad G_{ij}=hC_i^{-}b_j+c_{ij}-\frac{\varepsilon\mu-1}{\varepsilon\mu}d_{ij}+\delta_{ij}c_{\gamma\gamma}$$

$$L_{ij}=C_i^{+}c_j-c_{ij}-\delta_{ij}c_{\gamma\gamma},\qquad N_{ij}=hC_i^{-}c_j-g_{ij}+\frac{\varepsilon\mu-1}{\varepsilon\mu}a_{ij}-\delta_{ij}g_{\gamma\gamma}$$

$$C_i^{+}=\frac{B_{0i}^{+}+B_{0i}^{-}}{2},\qquad C_i^{-}=\frac{B_{0i}^{+}-B_{0i}^{-}}{2}\qquad (i,j=\alpha,\beta,\gamma)$$

Substituting the values of the displacement components and the electromagnetic field from (4.1) and (4.5) into the electrodynamics equations, taking into account (4.6) and averaging over the thickness of the shell, as it is done in the theory of shells, we obtain the following three fundamental equations:

$$\frac{1}{A^2}\frac{\partial^2\varphi}{\partial\alpha^2}+\frac{1}{B^2}\frac{\partial^2\varphi}{\partial\beta^2}-\frac{\varepsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial t^2}-\frac{4\pi\sigma\mu}{c^2}\frac{\partial\varphi}{\partial t}=-\frac{4\pi\sigma\mu}{c^3}\frac{3}{2h^3}\left(l_\beta\frac{\partial^2 w}{\partial t^2}+\frac{l}{B}\frac{\partial^3 w}{\partial\beta\,\partial t^2}-\right.$$
$$\left.l_\gamma\frac{\partial^2 v}{\partial t^2}\right)+\frac{\varepsilon\mu-1}{\varepsilon\mu c}\frac{3}{2h^3}\left[\frac{1}{A^2}\frac{\partial}{\partial\alpha}\left(l_\beta\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)-\frac{1}{AB}\frac{\partial}{\partial\alpha}\left(l_\alpha\frac{\partial^2 w}{\partial\beta\,\partial t}\right)-\frac{\varepsilon\mu}{c^2}l_\beta\frac{\partial^3 w}{\partial t^3}\right]$$

$$\frac{1}{A^2}\frac{\partial^2\psi}{\partial\alpha^2}+\frac{1}{B^2}\frac{\partial^2\psi}{\partial\beta^2}-\frac{\varepsilon\mu}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{4\pi\sigma\mu}{c^2}\frac{\partial\psi}{\partial t}=\frac{4\pi\sigma\mu}{c^3}\frac{3}{2h^3}\left(l_\alpha\frac{\partial^2 w}{\partial t^2}+\right. \tag{5.2}$$
$$\left.\frac{l}{A}\frac{\partial^3 w}{\partial\alpha\,\partial t^2}-l_\gamma\frac{\partial^2 u}{\partial t^2}\right)+\frac{\varepsilon\mu-1}{\varepsilon\mu c}\frac{3}{2h^3}\left[\frac{1}{AB}\frac{\partial}{\partial\beta}\left(l_\beta\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)-\frac{1}{B^2}\frac{\partial}{\partial\beta}\left(l_\alpha\frac{\partial^2 w}{\partial\beta\,\partial t}\right)+\frac{\varepsilon\mu}{c^2}l_\alpha\frac{\partial^3 w}{\partial t^3}\right]$$

$$\frac{1}{A^2}\frac{\partial^2 f}{\partial\alpha^2}+\frac{1}{B^2}\frac{\partial^2 f}{\partial\beta^2}-\frac{\varepsilon\mu}{c^2}\frac{\partial^2 f}{\partial t^2}-\frac{4\pi\sigma\mu}{c^2}\frac{\partial f}{\partial t}=\frac{4\pi\sigma}{c^2}\frac{3}{2h^3}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(l_\gamma\frac{\partial u}{\partial t}\right)+\right.$$
$$\frac{1}{B}\frac{\partial}{\partial\beta}\left(l_\gamma\frac{\partial v}{\partial t}\right)-\frac{1}{A}\frac{\partial}{\partial\alpha}\left(l_\alpha\frac{\partial w}{\partial t}\right)-\frac{1}{B}\frac{\partial}{\partial\beta}\left(l_\beta\frac{\partial w}{\partial t}\right)-\frac{1}{A^2}\frac{\partial}{\partial\alpha}\left(l\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)-$$
$$\left.\frac{1}{B^2}\frac{\partial}{\partial\beta}\left(l\frac{\partial^2 w}{\partial\beta\,\partial t}\right)\right]-\frac{\varepsilon\mu-1}{c^2\mu}\frac{3}{2h^3}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(l_\alpha\frac{\partial^2 w}{\partial t^2}\right)+\frac{1}{B}\frac{\partial}{\partial\beta}\left(l_\beta\frac{\partial^2 w}{\partial t^2}\right)\right]$$
$$l_i=\int_{-h}^{h}\gamma a_i d\gamma,\quad l=\int_{-h}^{h}\gamma a d\gamma\qquad (i=\alpha,\beta,\gamma)$$

In addition to Eqs. (5.2) we also obtain the following conditions which establish the connection between the surface values of the components of the electromagnetic field strength:

$$\frac{1}{A}\frac{\partial\psi}{\partial\alpha}-\frac{1}{B}\frac{\partial\varphi}{\partial\beta}+\frac{\mu}{c}\frac{\partial f}{\partial t}=0$$

$$\frac{1}{A}\frac{\partial f}{\partial\alpha}+\frac{4\pi\sigma}{c}\left[\psi+\frac{1}{2hc}\left(b_\alpha\frac{\partial w}{\partial t}-b_\gamma\frac{\partial u}{\partial t}+\frac{c_\gamma}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)\right]+$$
$$\frac{\varepsilon}{c}\frac{\partial\psi}{\partial t}+\frac{\varepsilon\mu-1}{c^2\mu}b_\alpha\frac{\partial^2 w}{\partial t^2}=\frac{h_\alpha^+-h_\alpha^-}{2h}$$

$$\frac{1}{B}\frac{\partial f}{\partial\beta}-\frac{4\pi\sigma}{c}\left[\varphi+\frac{1}{2hc}\left(b_\gamma\frac{\partial v}{\partial t}-b_\beta\frac{\partial w}{\partial t}-\frac{c_\gamma}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right)\right]-$$
$$\frac{\varepsilon}{c}\frac{\partial\varphi}{\partial t}+\frac{\varepsilon\mu-1}{c^2\mu}b_\beta\frac{\partial^2 w}{\partial t^2}=\frac{h_\beta^+-h_\beta^-}{2h}$$

$$\frac{1}{A}\frac{\partial\varphi}{\partial\alpha}+\frac{1}{B}\frac{\partial\psi}{\partial\beta}+\frac{\varepsilon\mu-1}{2h\varepsilon\mu c}\left(\frac{b_\beta}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}-\frac{b_\alpha}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right)=-\frac{e_\gamma^+-e_\gamma^-}{2h}$$

$$\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{h_\beta^++h_\beta^-}{2}\right)-\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{h_\alpha^++h_\alpha^-}{2}\right)=\frac{4\pi\sigma}{c}\frac{e_\gamma^++e_\gamma^-}{2}+\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left(\frac{e_\gamma^++e_\gamma^-}{2}\right)+$$
$$\frac{4\pi\sigma}{2hc^2}\left[b_\beta\frac{\partial u}{\partial t}-\frac{1}{B}\frac{\partial}{\partial\beta}\left(n_\gamma\frac{\partial u}{\partial t}\right)-b_\alpha\frac{\partial v}{\partial t}+\frac{1}{A}\frac{\partial}{\partial\alpha}\left(n_\alpha\frac{\partial v}{\partial t}\right)+\right.$$
$$\left.\left(\frac{1}{B}-\frac{\partial n}{\partial\beta}\frac{n_\beta}{\varepsilon\mu}-c_\beta\right)\frac{1}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}+\left(c_\alpha-\frac{1}{A}\frac{\partial n}{\partial\alpha}+\frac{n_\alpha}{\varepsilon\mu}\right)\frac{1}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right]$$

$$\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{h_\alpha^++h_\alpha^-}{2}\right)+\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{h_\beta^++h_\beta^-}{2}\right)+\frac{4\pi\sigma}{2hc^2}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(n_\alpha\frac{\partial w}{\partial t}\right)+\right.$$
$$\frac{1}{B}\frac{\partial}{\partial\beta}\left(n_\beta\frac{\partial w}{\partial t}\right)+\frac{1}{A^2}\frac{\partial}{\partial\alpha}\left(n\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)+\frac{1}{B^2}\frac{\partial}{\partial\beta}\left(n\frac{\partial^2 w}{\partial\beta\,\partial t}\right)-\frac{1}{A}\frac{\partial}{\partial\alpha}\left(n_\gamma\frac{\partial u}{\partial t}\right)-$$
$$\left.\frac{1}{B}\frac{\partial}{\partial\beta}\left(n_\gamma\frac{\partial v}{\partial t}\right)\right]+\frac{\varepsilon\mu-1}{2hc^2\mu}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(n_\alpha\frac{\partial^2 w}{\partial t^2}\right)+\frac{1}{B}\frac{\partial}{\partial B}\left(n_\beta\frac{\partial^2 w}{\partial t^2}\right)\right]=0$$

$$\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{e_\gamma^++e_\gamma^-}{2}\right)=\frac{\mu}{c}\frac{\partial}{\partial t}\left(\frac{h_\beta^++h_\beta^-}{2}\right)+\frac{4\pi\sigma\mu}{2hc^3}\left(n_\beta\frac{\partial^2 w}{\partial t^2}+\frac{n}{B}\frac{\partial^3 w}{\partial\beta\,\partial t^2}-\right.$$
$$\left.n_\gamma\frac{\partial^2 v}{\partial t^2}\right)+\frac{\varepsilon\mu-1}{2h\varepsilon\mu c}\left[\frac{\varepsilon\mu}{c^2}n_\beta\frac{\partial^3 w}{\partial t^3}-\frac{1}{A^2}\frac{\partial}{\partial\alpha}\left(n_\beta\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)+\frac{1}{AB}\frac{\partial}{\partial\alpha}\left(n_\alpha\frac{\partial^2 w}{\partial\beta\,\partial t}\right)\right]\quad(5.3)$$

$$\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{e_\gamma^++e_\gamma^-}{2}\right)=-\frac{\mu}{c}\frac{\partial}{\partial t}\left(\frac{h_\alpha^++h_\alpha^-}{2}\right)-\frac{4\pi\sigma\mu}{2hc^3}\left(n_\alpha\frac{\partial^2 w}{\partial t^2}+\right.$$
$$\left.\frac{n}{A}\frac{\partial^3 w}{\partial\alpha\,\partial t^2}-n_\gamma\frac{\partial^2 u}{\partial t^2}\right)-$$

$$\frac{\varepsilon\mu-1}{2h\varepsilon\mu c}\left[\frac{\varepsilon\mu}{c^2}n_\gamma\frac{\partial^3 w}{\partial t^3}-\frac{1}{AB}\frac{\partial}{\partial\beta}\left(n_\beta\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)+\frac{1}{B^2}\frac{\partial}{\partial\beta}\left(n_\alpha\frac{\partial^2 w}{\partial\beta\,\partial t}\right)\right]$$

$$n_i=\int_{-h}^{h}a_i d\gamma,\quad n=\int_{-h}^{h}a d\gamma \qquad (i=\alpha,\beta,\gamma)$$

Equations (5.1) and (5.2) form a complete system of six equations with respect to the six unknown functions. It should be noted here that the system of equations (5.1), (5.2) does not contain explicitly the elements of the solution of the exterior problem; this is explained by the acceptance of the fundamental hypothesis.

The relation between the interior and the exterior problems is realized through the boundary conditions for the unknown functions φ, ψ, f at the ends of the shell. We note that in some particular cases we can have conditions at the ends of the shell for which the interior problem is completely separated from the exterior one. We also note that the conditions (5.3) must be used as boundary conditions for the solution of the initial equations (1.4) of the exterior problem.

In order to solve concrete boundary value problems, we have to adjoin to the resolving differential equations (5.1), (5.2), the boundary conditions for the components of the electromagnetic field, as well as the usual conditions regarding the fixing of the edges of the shell.

The conditions for the components of the electromagnetic field are obtained from (1.6) where we take into account (1.5) and (1.7) and where $\mathbf{n}$ is the normal to the corresponding end surface. For the end surface $\alpha=\text{const}$ we have

$$\mathbf{n}=\mathbf{i}-\frac{1}{B}\left(\frac{\partial u}{\partial\beta}-\frac{\gamma}{A}\frac{\partial^2 w}{\partial\alpha\partial\beta}\right)\mathbf{j}+\frac{1}{A}\frac{\partial w}{\partial\alpha}\mathbf{k},\qquad v_n=\frac{\partial u_\alpha}{\partial t}$$

Therefore the linearized boundary conditions can be written in the following form:

$$h_\alpha=\frac{1}{\mu}h_\alpha^{(e)}+\frac{\mu-1}{\mu}B_{0\beta}^{(e)}\frac{1}{B}\left(\frac{\partial u}{\partial\beta}-\frac{\gamma}{A}\frac{\partial^2 w}{\partial\alpha\,\partial\beta}\right)-\frac{\mu-1}{\mu}\frac{B_{0\gamma}^{(e)}}{A}\frac{\partial w}{\partial\alpha}$$

$$h_\beta=h_\beta^{(e)}+\frac{\mu-1}{\mu}\frac{B_{0\alpha}^{(e)}}{B}\left(\frac{\partial u}{\partial\beta}-\frac{\gamma}{A}\frac{\partial^2 w}{\partial\alpha\,\partial\beta}\right),\qquad h_\gamma=h_\gamma^{(e)}-\frac{\mu-1}{\mu}\frac{B_{0\alpha}^{(e)}}{A}\frac{\partial w}{\partial\alpha}$$

$$e_\alpha=\frac{1}{\varepsilon}e_\alpha^{(e)},\qquad e_\beta=e_\beta^{(e)}+\frac{\mu-1}{c}B_{0\gamma}^{(e)}\left(\frac{\partial u}{\partial t}-\frac{\gamma}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)\tag{5.4}$$

$$e_\gamma=e_\gamma^{(e)}-\frac{\mu-1}{c}B_{0\beta}^{(e)}\left(\frac{\partial u}{\partial t}-\frac{\gamma}{A}\frac{\partial^2 w}{\partial\alpha\,\partial t}\right)$$

In a similar manner one can write the conditions at the end surface $\beta=\text{const}$. In the special case when the end surfaces are in vacuum, it is advisable to make use of condition (4.7), since this condition does not contain elements of the solution of the exterior problem. Then, for example, we have for the end surface $\alpha=\text{const}$

$$e_\alpha=\frac{1}{c}\left[B_{0\beta}\frac{\partial w}{\partial t}-B_{0\gamma}\left(\frac{\partial v}{\partial t}-\frac{\gamma}{B}\frac{\partial^2 w}{\partial\beta\,\partial t}\right)\right]\tag{5.5}$$

In the case when the shell is fixed along the considered edge with a resting ideal conductor, the interior problem is solved independently of the exterior one. This is explained by the fact that in a fixed ideal conductor the electric field is absent, i.e. the exterior electric field is equal to zero ($e_\alpha^{(e)} = e_\beta^{(e)} = e_\gamma^{(e)} = 0$). Then, the boundary conditions (5.4) for the components of the electric field obtain the form

$$e_\alpha = 0, \quad e_\beta = \frac{\mu - 1}{c} B_{0\gamma}^{(e)} \left(\frac{\partial u}{\partial t} - \frac{\gamma}{A} \frac{\partial^2 w}{\partial \alpha \partial t} \right)$$
$$e_\gamma = -\frac{\mu - 1}{c} B_{0\beta}^{(e)} \left(\frac{\partial u}{\partial t} - \frac{\gamma}{A} \frac{\partial^2 w}{\partial \alpha \partial t} \right) \tag{5.6}$$

Thus, examining the resolving equations (5.1), (5.2) which describe the oscillations of the shell, we note that they do not contain the components of the exterior induced electromagnetic field. The connection with the exterior domain (with the exterior problem) is accomplished only through the boundary conditions at the ends. In particular, when the boundary conditions do not contain the elements of the solution of the exterior problem, for example the conditions (5.6), the problem of the oscillation of the shell is solved independently of the exterior problem.

As an example we give some variants of boundary conditions (necessary for solving the system of equations (5.1), (5.2)) in the case when the shell is in contact at the edges with an ideal conductor. We give the conditions for the edge $\alpha =$ const.

Clamped edge

$$u = 0, \quad v = 0, \quad w = 0, \; \partial w/\partial \alpha = 0, \; \varphi = 0, \quad \psi = 0$$

Fixed-hinged edge

$$u = 0, \quad v = 0, \quad \partial^2 w/\partial \alpha^2 = 0, \; \varphi = 0, \; \psi = 0$$

Hinged-supported edge

$$v = 0, \quad w = 0, \quad \partial u/\partial \alpha = 0, \quad \partial^2 w/\partial \alpha^2 = 0 \quad \varphi = 0, \quad \psi = \frac{\mu - 1}{c} B_{0\gamma}^{(e)} \frac{\partial u}{\partial t}$$

In a similar way we can write down the boundary conditions for the edge $\beta =$ const.

6. For some particular cases of the exterior magnetic field, the resolving equations (5.1), (5.2) for concrete types of shell and plates become entirely suitable for solving concrete problems. We consider some of them.

Plate with a constant exterior magnetic field

$$D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} = P + \frac{2h^3\sigma}{3c}\left(B_{0\beta} \frac{\partial^2 \varphi}{\partial \alpha^2} - B_{0\alpha} \frac{\partial^2 \varphi}{\partial \alpha \partial \beta} + B_{0\beta} \frac{\partial^2 \psi}{\partial \alpha \partial \beta} - B_{0\alpha} \frac{\partial^2 \psi}{\partial \beta^2} \right) +$$
$$\frac{2h^3\sigma}{3c^2} \frac{\partial}{\partial t} \left[\left(\frac{1}{\varepsilon\mu} B_{0\beta}^2 + B_{0\gamma}^2 \right) \frac{\partial^2 w}{\partial \alpha^2} + \left(\frac{1}{\varepsilon\mu} B_{0\alpha}^2 + B_{0\gamma}^2 \right) \frac{\partial^2 w}{\partial \beta^2} - \frac{2}{\varepsilon\mu} B_{0\alpha} B_{0\beta} \frac{\partial^2 w}{\partial \alpha \partial \beta} \right] +$$
$$\frac{2h\sigma}{c} \left[B_{0\beta} \left(\varphi - \frac{B_{0\beta}}{c} \frac{\partial w}{\partial t} \right) - B_{0\alpha} \left(\psi + \frac{B_{0\alpha}}{c} \frac{\partial w}{\partial t} \right) \right]$$

$$\Delta\varphi - \frac{\varepsilon\mu}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2} \frac{\partial \varphi}{\partial t} = -\frac{\mu}{c} \frac{\partial}{\partial t} \left[\frac{4\pi\sigma}{c^2} B_{0\beta} \frac{\partial w}{\partial t} - \frac{\varepsilon\mu - 1}{\varepsilon\mu} \left(B_{0\beta} \frac{\partial^2 w}{\partial \alpha^2} - \right.\right.$$
$$\left.\left. B_{0\alpha} \frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{\varepsilon\mu}{c^2} B_{0\beta} \frac{\partial^2 w}{\partial t^2} \right) \right]$$

$$\Delta\psi - \frac{\varepsilon\mu}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2} \frac{\partial \psi}{\partial t} = \frac{\mu}{c} \frac{\partial}{\partial t} \left[\frac{4\pi\sigma}{c^2} B_{0\alpha} \frac{\partial w}{\partial t} + \frac{\varepsilon\mu - 1}{\varepsilon\mu} \left(B_{0\beta} \frac{\partial^2 w}{\partial \alpha \partial \beta} - \right.\right.$$
$$\left.\left. B_{0\alpha} \frac{\partial^2 w}{\partial \beta^2} + \frac{\varepsilon\mu}{c^2} B_{0\alpha} \frac{\partial^2 w}{\partial t^2} \right) \right] \left(\Delta = \frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2} \right)$$

Circular cylindrical shell with radius of curvature R, situated in a constant exterior magnetic field whose intensity vector is parallel to the elements of the cylinder. The coordinates α and β are chosen such that the coefficients of the first quadratic form should have the values $A = 1,\ B = R$

$$\frac{\partial^2 u}{\partial\alpha^2} + \frac{1-\nu}{2R^2}\frac{\partial^2 u}{\partial\beta^2} + \frac{1+\nu}{2R}\frac{\partial^2 v}{\partial\alpha\,\partial\beta} + \frac{\nu}{R}\frac{\partial w}{\partial\alpha} = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2}$$

$$\frac{1+\nu}{2R}\frac{\partial^2 u}{\partial\alpha\,\partial\beta} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial\alpha^2} + \frac{1}{R^2}\frac{\partial^2 v}{\partial\beta^2} + \frac{1}{R^2}\frac{\partial w}{\partial\beta} = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}$$

$$D\left[\frac{\partial^4 w}{\partial\alpha^4} + \frac{2}{R^2}\frac{\partial^4 w}{\partial\alpha^2\partial\beta^2} + \frac{1}{R^4}\frac{\partial^4 w}{\partial\beta^4} + \frac{3}{Rh^2}\left(\nu\frac{\partial u}{\partial\alpha} + \frac{1}{R}\frac{\partial v}{\partial\beta} + \frac{w}{R}\right)\right] +$$
$$2\rho h\frac{\partial^2 w}{\partial t^2} = P - \frac{2h^3\sigma}{3c}B_{0\alpha}\left(\frac{\partial^2\varphi}{\partial\alpha\partial\beta} + \frac{1}{R^2}\frac{\partial^2\psi}{\partial\beta^2}\right) + \frac{2h^3\sigma}{3c^2\varepsilon\mu R^2}\frac{\partial^3 w}{\partial\beta^2\partial t} -$$
$$\frac{2h\sigma}{c}B_{0\alpha}\left(\psi + \frac{B_{0\alpha}}{c}\frac{\partial w}{\partial t}\right)$$

$$\frac{\partial^2\varphi}{\partial\alpha^2} + \frac{1}{R^2}\frac{\partial^2\varphi}{\partial\beta^2} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2}\frac{\partial\varphi}{\partial t} = -\frac{\varepsilon\mu-1}{\varepsilon\mu c}\frac{B_{0\alpha}}{R}\frac{\partial^3 w}{\partial\alpha\partial\beta\partial t}$$

$$\frac{\partial^2\psi}{\partial\alpha^2} + \frac{1}{R^2}\frac{\partial^2\psi}{\partial\beta^2} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2}\frac{\partial\psi}{\partial t} = \frac{4\pi\sigma\mu}{c^3}B_{0\alpha}\frac{\partial^2 w}{\partial t^2} + \frac{\varepsilon\mu-1}{\varepsilon\mu c}B_{0\alpha}\frac{\partial}{\partial t}\times$$
$$\left(\frac{\varepsilon\mu}{c^2}\frac{\partial^2 w}{\partial t^2} - \frac{1}{R^2}\frac{\partial^2 w}{\partial\beta^2}\right)$$

Shallow shell with double curvature, situated in a constant exterior magnetic field whose intensity vector is perpendicular to the median surface of the shell. The coordinates α and β are chosen such that $A = 1,\ B = 1$

$$\frac{\partial^2 u}{\partial\alpha^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial\beta^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial\alpha\,\partial\beta} + (k_1 + \nu k_2)\frac{\partial w}{\partial\alpha} + \frac{1-\nu^2}{E}\frac{\sigma}{c}B_{0\gamma}\left(\psi - \frac{B_{0\gamma}}{c}\frac{\partial u}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 v}{\partial\beta^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial\alpha^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial\alpha\,\partial\beta} + (k_2 + \nu k_1)\frac{\partial w}{\partial\beta} - \frac{1-\nu^2}{E}\frac{\sigma}{c}B_{0\gamma}\left(\varphi + \frac{B_{0\gamma}}{c}\frac{\partial v}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}$$

$$D\left\{\Delta^2 w + \frac{3}{h^2}\left[(k_1+\nu k_2)\frac{\partial u}{\partial\alpha} + (k_2+\nu k_1)\frac{\partial v}{\partial\beta} + (k_1^2 + k_2^2 + 2\nu k_1 k_2)\,w\right]\right\} +$$
$$2\rho h\frac{\partial^2 w}{\partial t^2} = P + \frac{2h^3\sigma}{3c^2}B_{0\gamma}^2\frac{\partial\Delta w}{\partial t}$$

$$\Delta\varphi - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2}\frac{\partial\varphi}{\partial t} = \frac{4\pi\sigma\mu}{c^3}B_{0\gamma}\frac{\partial^2 v}{\partial t^2}$$

$$\Delta\psi - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{4\pi\sigma\mu}{c^2}\frac{\partial\psi}{\partial t} = -\frac{4\pi\sigma\mu}{c^3}B_{0\gamma}\frac{\partial^2 u}{\partial t^2}$$

Adjoining to the given resolving equations the necessary boundary conditions, we can find the solution of numerous magnetoelasticity problems for plates and shells. It is also necessary to keep in mind that in each of the considered problems the conditions (5.3) must be satisfied.

Thus, on the basis of the three-dimensional magnetoelasticity equations, a correct two-dimensional theory of shells and plates of finite conductivity has been constructed. This theory allows us to solve magnetoelasticity problems for shells and plates having finite dimensions.

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ON THE LOSS OF STABILITY OF NONSYMMETRIC STRICTLY CONVEX THIN SHALLOW SHELLS

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Values of the upper critical buckling loads of nonsymmetric strictly convex elastic shallow shells are determined when the relative wall thickness parameter is sufficiently small. Simple relationships are derived from which the mentioned values can be found if the character of the loading, the shell geometry, and the method of fixing the edge are known. In passing, asymptotic expansions of the solutions permitting a computation of the stress-strain state of shell in the precritical stage are constructed for the appropriate boundary value problems. As an